

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 Solve the inequality $2|3x - 1| < |x + 1|$.

[4]

Cambridge Math & Physics

kambridgemp.com

- Cambridge Math & Physics
- kambridgemp.com

- 3** (a) Given that $\cos(x - 30^\circ) = 2 \sin(x + 30^\circ)$, show that $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

bridge Math & Physics
www.bridgemp.com

- (b)** Hence solve the equation

$$\cos(x - 30^\circ) = 2 \sin(x + 30^\circ),$$

for $0^\circ < x < 360^\circ$.

[2]

[illegible]

- 4 (a) Prove that $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1 - \cos 2\theta}{1 + \cos 2\theta} d\theta$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 5 (a) Solve the equation $z^2 - 2pz - q = 0$, where p and q are real constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

In an Argand diagram with origin O , the roots of this equation are represented by the distinct points A and B .

- (b) Given that A and B lie on the imaginary axis, find a relation between p and q . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Cambridge Math & Physics

kanbridgemp.com

6 The parametric equations of a curve are

$$x = \ln(2 + 3t), \quad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive.

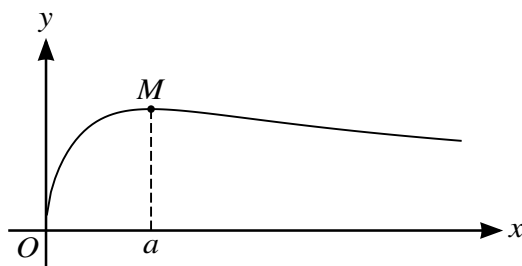
[5]

Cambridge Math & Physics

kambridgemp.com

Kambridge Math & Physics
kcambridgemp.com

7



The diagram shows the curve $y = \frac{\tan^{-1} x}{\sqrt{x}}$ and its maximum point M where $x = a$.

(a) Show that a satisfies the equation

$$a = \tan\left(\frac{2a}{1+a^2}\right). \quad [4]$$

Kambridge Math & Physics
kambridgemp.com

- (b) Verify by calculation that a lies between 1.3 and 1.5.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

- (a) Find the acute angle between the directions of AB and l . [4]

Kambridge Math & Physics
kambridgemp.com

[5]

Cambridge Math & Physics

kanbridgemp.com

- 9** The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point.

[5]

Cambridge Math & Physics

kambridgemp.com

(b) Show that $\int_1^8 y \, dx = 18 \ln 2 - 9$.

[5]

Cambridge Math & Physics

- 10** The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x .

[11]

Cambridge Math & Physics
kamdridgemp.com

Kambridge Math & Physics
kambridgemp.com

Kambridge Math & Physics
kambridgeemp.com

Kambridge Math & Physics
kambridgemp.com

Kambridge Math & Physics
kambridgemp.com

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.