

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- [4]

Kambridge Math & Physics

2 Find the exact value of $\int_0^1 (2-x)e^{-2x} dx$.

[5]

Kambridge Math & Physics
kambridgemp.com

The image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. A large, light gray watermark is overlaid diagonally from the bottom left towards the top right. The watermark consists of the text "Kambridge Math & Physics" in a bold, sans-serif font, followed by the website address "kambridgemp.com" in a smaller, regular font below it.

- 3 (a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x .

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 The equation of a curve is $y = x \tan^{-1}(\frac{1}{2}x)$.

- (a) Find $\frac{dy}{dx}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) The tangent to the curve at the point where $x = 2$ meets the y-axis at the point with coordinates $(0, p)$.

Find p . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 By first expressing the equation

$$\tan \theta \tan(\theta + 45^\circ) = 2 \cot 2\theta$$

as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 90^\circ$.

[6]

Cambridge Math & Physics

kambridgemp.com

Kambridge Math & Physics
kambridgemp.com

- 6 (a) By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

- (b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part (a). [2]

- (c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

7 Let $f(x) = \frac{2}{(2x-1)(2x+1)}$.

(a) Express $f(x)$ in partial fractions.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Using your answer to part (a), show that

$$(f(x))^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}. \quad [2]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

Kambridge Math & Physics

kambridgemp.com

- 8** Relative to the origin O , the points A , B and D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{k}.$$

A fourth point C is such that $ABCD$ is a parallelogram.

- (a) Find the position vector of C and verify that the parallelogram is not a rhombus. [5]

Cambridge Math & Physics
kambriqemp.com

- (b) Find angle BAD , giving your answer in degrees.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the area of the parallelogram correct to 3 significant figures.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 9 (a)** The complex numbers u and w are such that

$$u - w = 2i \quad \text{and} \quad uw = 6.$$

Find u and w , giving your answers in the form $x + iy$, where x and y are real and exact. [5]

Cambridge Math & Physics

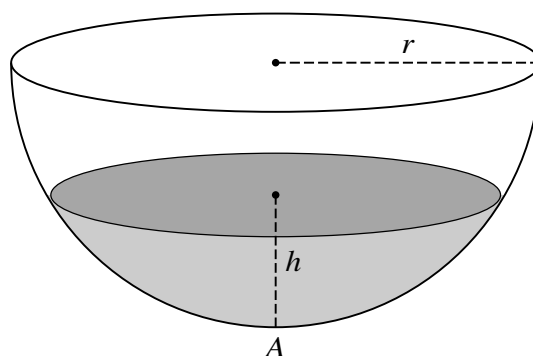
kambridgemp.com

- (b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2i| \leq 2, \quad 0 \leq \arg z \leq \frac{1}{4}\pi \quad \text{and} \quad \operatorname{Re} z \leq 3. \quad [5]$$

Kambridge Math & Physics
kambridgemp.com

10



A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r , as shown in the diagram. The depth of water at time t is h . At time $t = 0$ the tank is full and the depth of the water is r . At this instant a tap at A is opened and water begins to flow out at a rate proportional to $\sqrt{h}$. The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h . It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant.

[4]

Kambridge Math & Physics
kambridgeemp.com

- (b)** Solve the differential equation and obtain an expression for t in terms of h and r . [8]

Cambridge Math & Physics

kambridgemp.com

Kambridge Math & Physics
kambridgeemp.com

Kambridge Math & Physics
kambridgemp.com

Kambridge Math & Physics
kambridgemp.com

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.