

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

Cambridge Math & Physics

kambridgemp.com

- 2 (a)** Expand $(2 - 3x)^{-2}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

Cambridge Math & Physics
kambriqemp.com

- (b) State the set of values of x for which the expansion is valid. [1]

.....

.....

.....

.....

.....

- 3** Express the equation $\tan(\theta + 60^\circ) = 2 + \tan(60^\circ - \theta)$ as a quadratic equation in $\tan \theta$, and hence solve the equation for $0^\circ \leq \theta \leq 180^\circ$. [6]

Kambridge Math & Physics
kambridgemp.com

- 4** The curve with equation $y = e^{2x}(\sin x + 3 \cos x)$ has a stationary point in the interval $0 \leq x \leq \pi$.

(a) Find the x -coordinate of this point, giving your answer correct to 2 decimal places. [4]

Cambridge Math & Physics
kamdridgemp.com

(b) Determine whether the stationary point is a maximum or a minimum. [2]

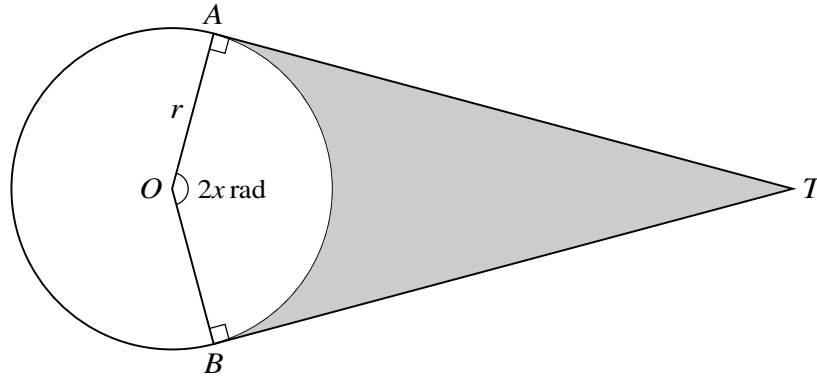
[illegible]

- 5** (a) Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

Cambridge Math & Physics

kambridgemp.com

Cambridge Math & Physics
kamdridgemp.com



The diagram shows a circle with centre O and radius r . The tangents to the circle at the points A and B meet at T , and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT , and by the minor arc AB . The area of the shaded region is equal to the area of the circle.

- (a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

Cambridge Math & Prep
cambridgeprep.com

- (b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 Let $f(x) = \frac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

Cambridge Math & Physics

kambridgemp.com

- (b) Find $\int_{\frac{1}{2}\pi}^{\frac{1}{2}\pi} f(x) dx$. Give your answer in a simplified exact form. [4]

Cambridge Math & Physics
kamdridgemp.com

- (a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [8]

Kambridge Math & Physics
kambridgemp.com

(b) Describe what happens to y as x tends to infinity.

[1]

- 9 With respect to the origin O , the vertices of a triangle ABC have position vectors

$$\vec{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \vec{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \vec{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

- (a) Using a scalar product, show that angle ABC is a right angle. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Show that triangle ABC is isosceles. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Kambridge Math & Physics

kambridgemp.com

10 (a) The complex number u is defined by $u = \frac{3i}{a + 2i}$, where a is real.

(i) Express u in the Cartesian form $x + iy$, where x and y are in terms of a . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) (i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leq |z - 1 - i|$ and $|z - 2 - i| \leq 2$. [4]

- (ii) Calculate the least value of $\arg z$ for points in this region. [2]

[illegible]

Kambridge Math & Physics
kambridgeemp.com

Kambridge Math & Physics
kambridgemp.com

Kambridge Math & Physics
kambridgemp.com

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.