Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 An arithmetic progression has fourth term 15 and eighth term 25.

Find the 30th term of the progression. [3]

2 Find the exact solution of the equation

$$\cos\frac{1}{6}\pi + \tan 2x + \frac{\sqrt{3}}{2} = 0 \text{ for } -\frac{1}{4}\pi < x < \frac{1}{4}\pi. \quad [2]$$

3 **(a)** Find the coefficients of x^3 and x^4 in the expansion of $(3-ax)^5$, where a is a constant. Give your answers in terms of a. [3]

(b) Given that the coefficient of x^4 in the expansion of $(ax+7)(3-ax)^5$ is 240, find the positive value of a. [3]

4 Solve the equation $4\sin^4\theta + 12\sin^2\theta - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

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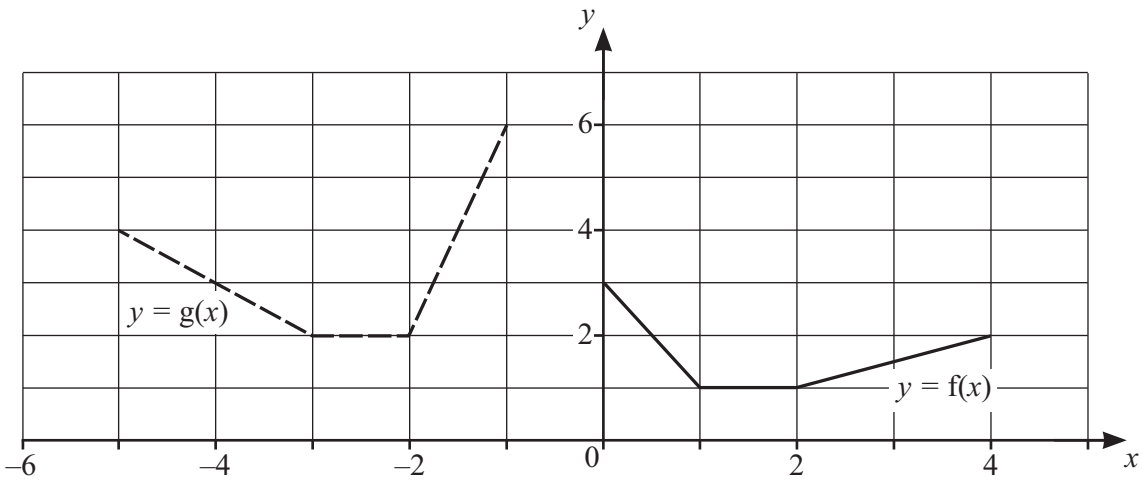

In the diagram, the graph with equation $y = f(x)$ is shown with solid lines and the graph with equation $y = g(x)$ is shown with broken lines.

(a) Describe fully a sequence of three transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [6]

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(b) Find an expression for $g(x)$ in the form $af(bx+c)$, where a, b and c are integers. [2]

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6 The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

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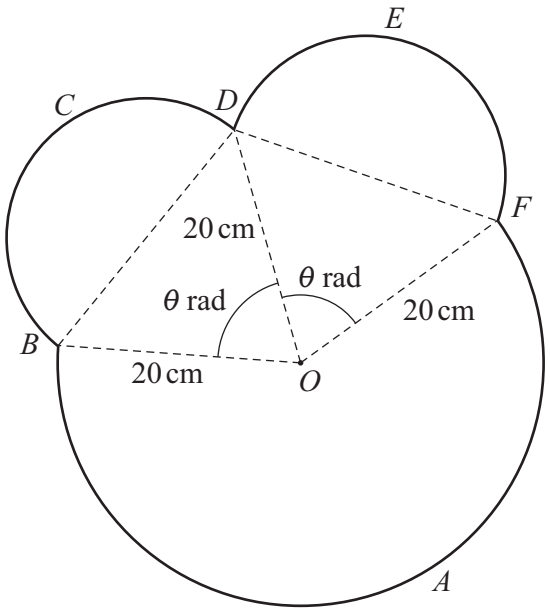

The diagram shows a metal plate *ABCDEF* consisting of five parts. The parts *BCD* and *DEF* are semicircles. The part *BAFO* is a sector of a circle with centre *O* and radius 20 cm, and *D* lies on this circle. The parts *OBD* and *ODF* are triangles. Angles *BOD* and *DOF* are both θ radians.

(a) Given that $\theta = 1.2$, find the area of the metal plate. Give your answer correct to 3 significant figures. [5]

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(b) Given instead that the area of each semicircle is $50\pi\,\text{cm}^2$, find the exact perimeter of the metal plate. [5]

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8 **(a)** Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

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The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

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For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

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(d) Hence or otherwise solve the equation $\mathrm{ff}(x) = 29$. [3]

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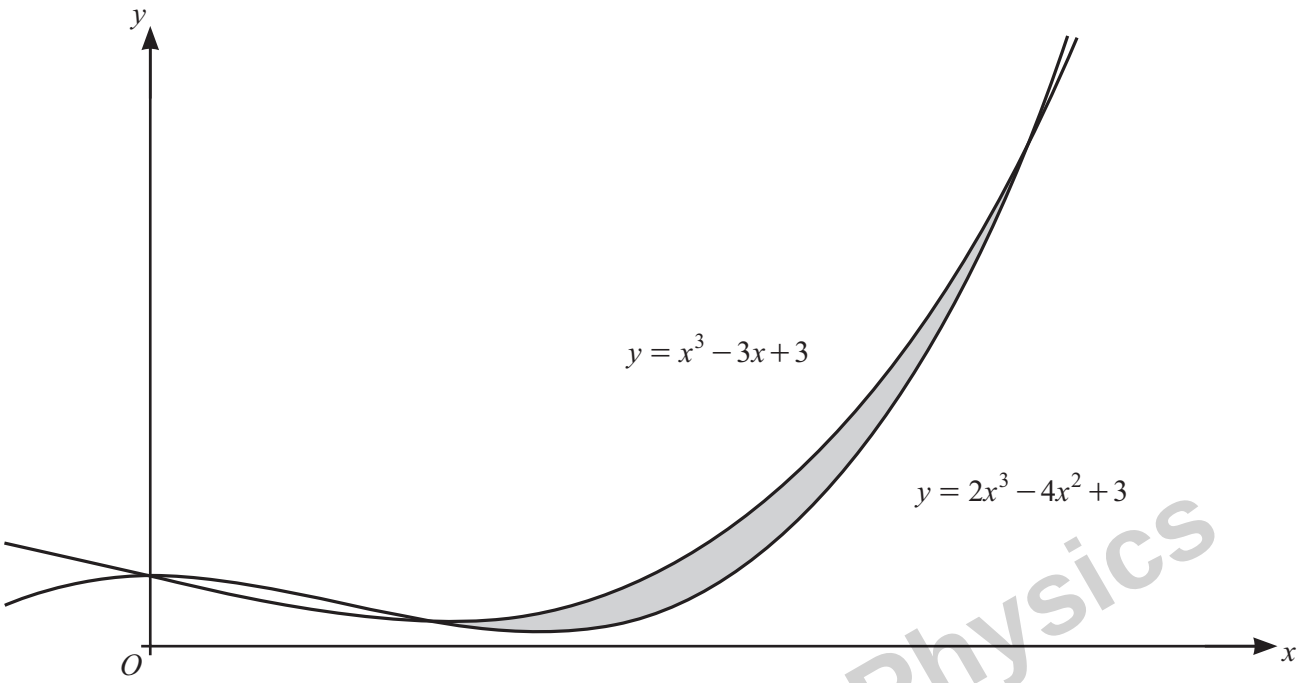

The diagram shows the curves with equations $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3$.

(a) Find the x-coordinates of the points of intersection of the curves. [3]

(b) Find the area of the shaded region. [4]

10 Points A and B have coordinates (4, 3) and (8, −5) respectively. A circle with radius 10 passes through the points A and B.

(a) Show that the centre of the circle lies on the line $y = \frac{1}{2}x - 4$. [4]

(b) Find the two possible equations of the circle. [5]

11 The equation of a curve is $y = kx^{\frac{1}{2}} - 4x^2 + 2$, where k is a constant.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of k. [2]

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(b) It is given that $k = 2$.

Find the coordinates of the stationary point and determine its nature. [4]

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(c) Points A and B on the curve have x-coordinates 0.25 and 1 respectively. For a different value of k, the tangents to the curve at the points A and B meet at a point with x-coordinate 0.6.

Find this value of k. [6]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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