Cambridge International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Integrate to obtain $-2x^{-1}$	B1	OE
	Substitute limits correctly with clear indication seen that upper limit gives 0	M1	For integral of form $-k\,x^{-n}$, where $k>0$, $n>0$.
	Obtain $\frac{2}{3}$	A1	WWW Accept 0.667.
		3	

Question	Answer	Marks	Guidance
2(a)	State $(3\pi,\ -k)$	B1	
		1	
2(b)	Obtain equation of form $[y=]c\pm k\sin\frac{1}{2}x$	M1	Any non-zero *c*.
	Obtain correct equation $[y=]2-k\sin\frac{1}{2}x$	A1	OE
	State $(3\pi,\ 2+k)$	B1 FT	Following part **(a)**, i.e. (*their x*, 2 – *their y*).
		3	

Question	Answer	Marks	Guidance
3	Integrate to obtain form $k(4x+5)^{\frac{3}{2}}$	*M1	
	Obtain correct $\frac{1}{2}(4x+5)^{\frac{3}{2}}$	A1	Or (unsimplified) equivalent. Condone missing … $+c$ so far.
	Substitute $x=1,\ y=9$ to form an equation in c	DM1	
	Obtain or imply $[y=]\ \dfrac{1}{2}(4x+5)^{\frac{3}{2}}-\dfrac{9}{2}$	A1	May be implied by $[a=]\dfrac{1}{2}(4(1)+5)^{\frac{3}{2}}-\dfrac{9}{2}$.
	Substitute $x=5$ to obtain $a=58$	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	Expand bracket to obtain 3 terms and use correct identity	M1	θ may be missing or another symbol used.
	Use identity $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	M1	Does not require any further explanation. θ may be missing or another symbol used.
	Conclude with $2\tan\theta$	A1	WWW AG
		3	

Question	Answer	Marks	Guidance
4(b)	Attempt solution of $5\tan^3\theta = 2\tan\theta$ to obtain at least one value of $\tan\theta$	M1	SOI Can be awarded if $\tan\theta$ is cancelled and ignored.
	Obtain at least two of $0, \pm 32.3$	A1	Or greater accuracy. **SC B1** if no method shown.
	Obtain all three values	A1	Or greater accuracy; and no others in $-90° < \theta < 90°$ range. Other units **SC B1** only for all 3 angles. **SC B1** if no method shown.
		3	

Question	Answer	Marks	Guidance
5	Differentiate to obtain form $kx(2x^2-5)^{-2}$	M1	
	Obtain correct $-12x(2x^2-5)^{-2}$	A1	OE
	Substitute (2, 1) to obtain gradient $-\frac{24}{9}$	A1	OE e.g. $-\dfrac{8}{3}$. Allow -2.67.
	Apply negative reciprocal to *their* numerical gradient to obtain gradient of normal	*M1	Must have been some attempt at differentiation. Expect $\dfrac{3}{8}$
	Attempt equation of normal using *their* gradient of the normal and (2, 1)	DM1	Expect $y-1=\dfrac{3}{8}(x-2)$.
	Obtain $3x-8y+2=0$ (allow multiples)	A1	Or equivalent of requested form e.g. $8y-3x-2=0$.
		6	

Question	Answer	Marks	Guidance
6	$\binom{4}{2}2^2(ax)^2,\ \binom{4}{3}2^{[1]}(ax)^3$	**B1 B1**	OE Expect $24a^2x^2, 8a^3x^3$ (may be seen in an expansion).
	Multiply terms involving x^2 and x^3 by $5-ax$ to obtain x^3 term	***M1**	Must find two products only (may be seen in an expansion).
	Equate coefficient of x^3 to 432 and solve for a	**DM1**	Ignore inclusion of x^3 at this stage.
	Obtain $a=3$ only	**A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	Attempt substitution for y in quadratic equation	***M1**	Or substitution for x …
	Obtain $5x^2+30x+75-k[=0]$ or $5y^2-20y+50-k[=0]$	**A1**	OE e.g. $x^2+6x+15-\frac{k}{5}$ (all terms gathered together).
	Use $b^2-4ac=0$ with *their a*, *b* and *c*	**DM1**	‘ = 0’ may be implied in subsequent working or the answer.
	Obtain $900-20(75-k)=0$ or equivalent and hence $k=30$	**A1**	… obtaining $400-20(50-k)=0$ and $k=30$.
		4	
7(b)	Substitute *their* value of k in equation from part **(a)** and attempt solution	**M1**	Expect $5x^2+30x+45[=0]$ or $5y^2-20y+20[=0]$.
	Obtain coordinates $(-3,\ 2)$	**A1**	**SC B1** only $(-3,\ 2)$ without attempt at quadratic solution.
		2	

Question	Answer	Marks	Guidance
8(a)	Substitute $n=10$ and $a=6$ into $u_n = a+(n-1)d$	*M1	Expect $6+9d=19.5$ or equivalent.
	$[d=]1.5$	A1	
	Substitute $a=6$ and *their d* into correct formula for the sum of 100 terms	DM1	
	Obtain 8025	A1	
		4	
8(b)	Obtain $S=48$	B1	
	Identify for S_E first term 12 and common ratio $\frac{1}{4}$	B1	
	Attempt sum to infinity, S_E, with at least one of first term and common ratio correct	M1	Only awarded if $\|r\| < 1$.
	Obtain $S_E = 16$	A1	
		4	

Question	Answer	Marks	Guidance
9(a)	Attempt to form expression for $\mathrm{gf}(x)$	*M1	Expect $5\left((3x-2)^2+k\right)-1$; fg($x$) is M0. Do not allow algebraic errors.
	Obtain $5(3x-2)^2+5k-1$	A1	OE e.g. $45x^2-60x+5k+19$.
	Their $5k-1=39$ or $5k-1\geqslant 39$	DM1	Or use $b^2-4ac=0$ (must be '= 0', could be implied later) on $45x^2-60x+5k+19-39\geqslant 0$ OE.
	Obtain $k=8$	A1	Do not accept $k\geqslant 8$.
		4	
9(b)	Obtaining $(3(5x-1)-2)^2+\mathit{their}\,k$	M1	May simplify and/or use k at this stage; k may have come from an inequality in **(a)**.
	Conclude $[\mathrm{fg}(x)]\geqslant 8$ allow $[y]\geqslant 8$	A1 FT	OE Following *their* value of k; must be ⩾, not >. Allow an accurate written description.
		2	
9(c)	State $\mathrm{g}^{-1}(x)=\frac{1}{5}(x+1)$	B1	OE $\frac{1}{5}(x+1)$ must be indicated as the inverse.
	$[\mathrm{h}(x)=]7x+4$	B1B1	If $7x+4$ only, it must be clear that this is $\mathrm{h}(x)$.
		3	

Question	Answer	Marks	Guidance
10(a)	Obtain gradient of relevant radius is –2	**B1**	
	Using $m_1 m_2 = -1$ obtain the gradient of the tangent and use it to form a straight line equation for a line containing (–6, 9)	**M1**	m_1 must be from an attempt to find the gradient of the radius using the centre and the given point.
	Obtain $y = \frac{1}{2}x + 12$	**A1**	OE e.g. $y - 9 = \frac{1}{2}(x+6)$.
		3	
10(b)	State or imply $(x+4)^2 + (y-5)^2 = 20$	**B1**	If $x^2 + y^2 - 2gx - 2fy + c = 0$ is used correctly with $(-g, -f) = (-4, 5)$ and $c = g^2 + f^2 - r^2$ then M1.
	Obtain $x^2 + y^2 + 8x - 10y + 21 = 0$	**B1**	A1 if above method used.
		2	
10(c)	Substitute $x = 0$ in equation of circle to find y-values 3 and 7 or state C to $AB = 4$	**B1**	May be implied by $AB = 4$ or use of $\lvert x$-coordinate of $C\rvert$.
	Attempt value of θ either using cosine rule or via $\frac{1}{2}\theta$ using right-angled triangle	**M1**	Using *their* AB. If $\theta/2$ used, must be multiplied by 2.
	Obtain $\theta = 0.9273$	**A1**	Or greater accuracy. A correct answer implies the M1.
		3	

Question	Answer	Marks	Guidance
10(d)	Attempt arc length using $r\theta$ formula with *their* θ (not *their* $\theta/2$) and $r=\sqrt{20}$	**M1**	Expect 4.15.
	Obtain perimeter = 8.15 or greater accuracy	**A1**	Condone missing units or incorrect units.
	Attempt area using $\frac{1}{2}r^2(\theta-\sin\theta)$ formula or equivalent with *their* θ and $r=\sqrt{20}$	**M1**	If sector – triangle used, both formulae must be correct. If triangle ACM used, area must be multiplied by 2.
	Obtain area = 1.27 or greater accuracy	**A1**	Condone missing units or incorrect units.
		4	

Question	Answer	Marks	Guidance
11(a)	Differentiate to obtain $-\frac{4}{3}x^{-\frac{5}{3}}+x^{-\frac{4}{3}}$ **or** rewrite as a quadratic equation in $x^{-\frac{1}{3}}$ or $x^{\frac{1}{3}}$	**B1**	Expect quadratic $2\left(x^{-\frac{1}{3}}\right)^2-3x^{-\frac{1}{3}}+1$ OE Allow $2x^2-3x+1$.
	Equate first derivative to zero and reach a solution for $x^{-\frac{1}{3}}$ or $x^{\frac{1}{3}}$ with no error in use of indices **or** complete square to find minimum point $2\left(a-\frac{3}{4}\right)^2-\frac{1}{8}$ where $a=x^{-\frac{1}{3}}$	**M1**	Substitution SOI if dealt with correctly later
	Obtain $x=\frac{64}{27}$	**A1**	Or exact equivalent. **SC B1** if no working shown. Ignore extra solution $x=0$.
	$y=-\frac{1}{8}$ seen	**B1**	Or exact equivalent. Allow -0.125.
		4	

Question	Answer	Marks	Guidance
11(b)	Recognise equation as quadratic in $x^{-\frac{1}{3}}$ or equivalent and attempt solution	**M1**	$2a^2 - 3a + 1[=0]$ where $a = x^{-\frac{1}{3}}$.
	Obtain $x^{-\frac{1}{3}} = 1$ and $x^{-\frac{1}{3}} = \frac{1}{2}$	**A1**	OE **SC B1** if no M mark awarded.
	Obtain 1 and 8	**A1**	**SC B1** if no M mark awarded.
	Integrate to obtain form $k_1 x^{\frac{1}{3}} + k_2 x^{\frac{2}{3}} + x$ or 2 out of 3 correct terms	***M1**	Expect $6x^{\frac{1}{3}} - \frac{9}{2}x^{\frac{2}{3}} + x$.
	Obtain correct $6x^{\frac{1}{3}} - \frac{9}{2}x^{\frac{2}{3}} + x$	**A1**	No other terms from a second integral.
	Apply *their* limits correctly	**DM1**	*Their* limits must be from *their* working.
	[Obtain –0.5 and conclude area is] 0.5	**A1**	
		7	