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MATHEMATICS **9709/12**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	Coefficient of $x = 80(x)$	**B2**	Correct value must be selected for both marks. SR +80 seen in an expansion gets **B1** or −80 gets **B1** if selected.
	Total:	**2**	
1(ii)	Coefficient of $\frac{1}{x} = -40\left(\frac{1}{x}\right)$	**B2**	Correct value soi in (ii), if powers unsimplified only allow if selected. SR +40 soi in (ii) gets **B1**.
	Coefficient of x = (1 × their 80) + (3 ×their − 40) = −40(x)	**M1 A1**	Links the appropriate 2 terms only for **M1**.
	Total:	**4**	
2(i)	Gradient = 1.5 Gradient of perpendicular = $-\frac{2}{3}$	**B1**	
	Equation of AB is $y - 6 = -\frac{2}{3}(x + 2)$ Or $3y + 2x = 14$ oe	**M1 A1**	Correct use of straight line equation with a changed gradient and (− 2, 6), the (− (− 2)) must be resolved for the **A1** ISW.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	**3**	
2(ii)	Simultaneous equations → Midpoint (1, 4)	**M1**	Attempt at solution of simultaneous equations as far as $x =$, or $y =$.
	Use of midpoint or vectors → B (4, 2)	**M1A1**	Any valid method leading to x, or to y.
	Total:	**3**	

Question	Answer	Marks	Guidance
3(i)	LHS = $\left(\frac{1}{c}-\frac{s}{c}\right)^2$	**M1**	Eliminates tan by replacing with $\frac{\sin}{\cos}$ leading to a function of sin and/or cos only.
	$=\frac{(1-s)^2}{1-s^2}$	**M1**	Uses $s^2 + c^2 = 1$ leading to a function of sin only.
	$=\frac{(1-s)(1-s)}{(1-s)(1+s)}=\frac{1-\sin\theta}{1+\sin\theta}$	**A1**	AG. Must show use of factors for **A1**.
	Total:	**3**	
3(ii)	Uses part (i) → $2 - 2s = 1 + s$		
	→ $s = ⅓$	**M1**	Uses part (i) to obtain $s = k$
	$\theta = 19.5°$ or $160.5°$	**A1A1 FT**	FT from error in 19.5° Allow 0.340^c (0.3398^c) & $2.80(2)$ or $0.108\pi^c$ & $0.892\pi^c$ for **A1** only. Extra answers in the range lose the second **A1** if gained for 160.5°.
	Total:	**3**	
4(i)	$(AB) = 2r\sin\theta$ (or $r\sqrt{2-2\cos 2\theta}$ or $\frac{r\sin 2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	**B1**	Allow unsimplifed throughout eg r + r, $\frac{2\theta}{2}$ etc
	(Arc AB) = $2r\theta$	**B1**	
	$(P =)\ 2r + 2r\theta + 2r\sin\theta$ (or $r\sqrt{2-2\cos 2\theta}$ or $\frac{r\sin 2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	**B1**	
	Total:	**3**	

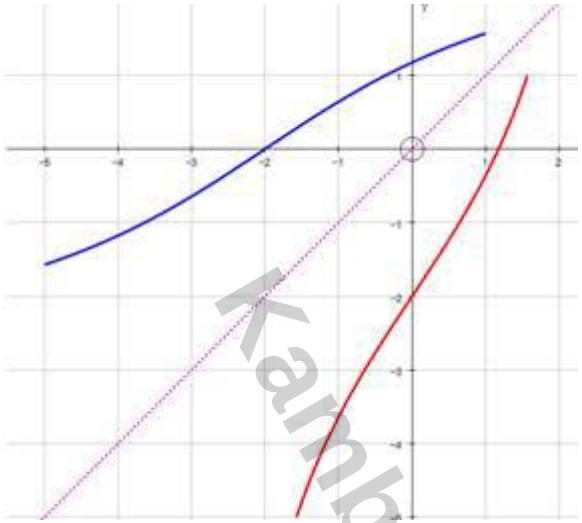
Question	Answer	Marks	Guidance
4(ii)	Area sector $AOB = (\frac{1}{2} r^2 2\theta)$ $\frac{25\pi}{6}$ or 13.1	**B1**	Use of segment formula gives 2.26 **B1B1**
	Area triangle $AOB = (\frac{1}{2} \times 2r\sin\theta \times r\cos\theta$ or $\frac{1}{2} \times r^2 \sin 2\theta)$ $\frac{25\sqrt{3}}{4}$ or 10.8	**B1**	
	Area rectangle $ABCD = (r \times 2r\sin\theta)\ 25$	**B1**	
	(Area =) Either $25 - (25\pi/6 - 25\sqrt{3}/4)$ or 22.7	**B1**	Correct final answer gets **B4**.
	Total:	**4**	
5(i)	Crosses x-axis at (6, 0)	**B1**	$x = 6$ is sufficient.
	$\frac{dy}{dx} = (0 +) -12\ (2 - x)^{-2} \times (-1)$	**B2,1,0**	−1 for each incorrect term of the three or addition of + C.
	Tangent $y = \frac{3}{4}(x - 6)$ or $4y = 3x - 18$	**M1 A1**	Must use dy/dx, x= their 6 but not $x = 0$ (which gives $m = 3$), and correct form of line equation.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	**5**	
5(ii)	If $x = 4$, $dy/dx = 3$		
	$\frac{dy}{dt} = 3 \times 0.04 = 0.12$	**M1 A1FT**	**M1** for ("their m" from $\frac{dy}{dx}$ and $x = 4$) × 0.04. Be aware: use of $x = 0$ gives the correct answer but gets **M0**.
	Total:	**2**	

Question	Answer	Marks	Guidance
6	$\text{Vol} = \pi \int (5-x)^2 \, dx - \pi \int \frac{16}{x^2} dx$	**M1***	Use of volume formula at least once, condone omission of π and limits and dx.
		DM1	Subtracting volumes somewhere must be <u>after</u> squaring.
	$\int (5-x)^2 dx = \frac{(5-x)^3}{3} \div -1$	**B1 B1**	**B1** Without ÷ (−1). **B1** for ÷(−1)
	(or $25x - 10x^2/2 + \frac{1}{3}x^3$)	**(B2,1,0)**	−1 for each incorrect term
	$\int \frac{16}{x^2} dx = -\frac{16}{x}$	**B1**	
	Use of limits 1 and 4 in an integrated expression and subtracted.	**DM1**	Must have used "y^2" at least once. Need to see values substituted.
	→ 9π or 28.3	**A1**	
	Total:	**7**	
7(a)	$(S_n =) \frac{n}{2}\left[32 + (n-1)8\right]$ and 20000	**M1**	**M1** correct formula used with d from $16 + d = 24$
		A1	**A1** for correct expression linked to 20000.
	→ $n^2 + 3n - 5000$ (<,=,> 0)	**DM1**	Simplification to a three term quadratic.
	→ ($n = 69.2$) → 70 terms needed.	**A1**	Condone use of 20001 throughout. Correct answer from trial and improvement gets 4/4.
	Total:	**4**	

Question	Answer	Marks	Guidance
7(b)	$a = 6,\ \frac{a}{1-r} = 18 \rightarrow r = ⅔$	**M1A1**	Correct $S\infty$ formula used to find r.
	New progression $a = 36,\ r = \frac{4}{9}$ oe	**M1**	Obtain new values for a and r by any valid method.
	New $S\infty = \frac{36}{1-\frac{4}{9}} \rightarrow 64.8$ or $\frac{324}{5}$ oe	**A1**	(Be aware that $r = -⅔$ leads to 64.8 but can only score M marks)
	Total:	**4**	
8(i)	Uses scalar product correctly: $3 \times 6 + 2 \times 6 + (-4) \times 3 = 18$	**M1**	Use of dot product with $\overrightarrow{OA}$ or $\overrightarrow{AO}$ & $\overrightarrow{OB}$ or $\overrightarrow{BO}$ only.
	$\lvert\overrightarrow{OA}\rvert = \sqrt{29}$, $\lvert\overrightarrow{OB}\rvert = 9$	**M1**	Correct method for any one of $\lvert\overrightarrow{OA}\rvert$, $\lvert\overrightarrow{AO}\rvert$, $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{BO}\rvert$.
	$\sqrt{29} \times 9 \times \cos AOB = 18$	**M1**	All linked correctly.
	$\rightarrow AOB = 68.2°$ or 1.19^c	**A1**	Multiples of π are acceptable (e.g. $0.379\pi^c$)
	Total:	**4**	
8(ii)	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + (3+2p)\mathbf{k}$	***M1**	For use of $\overrightarrow{OB} - \overrightarrow{OA}$ **,** allow with p = 2
	Comparing "**j**"	**DM1**	For comparing, $\overrightarrow{OC}$ must contain p & q. Can be implied by $\overrightarrow{AB} = 2\overrightarrow{OC}$.
	$\rightarrow p = 2½$ and $q = 4$	**A1 A1**	Accuracy marks only available if $\overrightarrow{AB}$ is correct.
	Total:	**4**	

Question	Answer	Marks	Guidance
9(i)	$\frac{dy}{dx} = 4x^{-½} - 2$	**B1**	Accept unsimplified.
	= 0 when $\sqrt{x} = 2$		
	$x = 4, y = 8$	**B1B1**	
	Total:	**3**	
9(ii)	$\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}}$	**B1FT**	FT providing –ve power of x
	$\left(\frac{d^2y}{dx^2} = -\frac{1}{4}\right) \rightarrow$ Maximum	**B1**	Correct $\frac{d^2y}{dx^2}$ and x=4 in (i) are required. Followed by"< 0 or negative" is sufficient" but $\frac{d^2y}{dx^2}$ must be correct if evaluated.
	Total:	**2**	
9(iii)	*EITHER:* Recognises a quadratic in $\sqrt{x}$	**(M1**	Eg $\sqrt{x} = u \rightarrow 2u^2 - 8u + 6 = 0$
	1 and 3 as solutions to this equation	**A1**	
	$\rightarrow x = 9, x = 1.$	**A1)**	

Question	Answer	Marks	Guidance
	OR: Rearranges then squares	**(M1**	$\sqrt{x}$ needs to be isolated before squaring both sides.
	$\rightarrow\ x^2 - 10x + 9 = 0$ oe	**A1**	
	$\rightarrow x = 9, x = 1.$	**A1)**	Both correct by trial and improvement gets 3/3
	Total:	**3**	
9(iv)	$k > 8$	**B1**	
	Total:	**1**	
10(i)	$3\tan\left(\frac{1}{2}x\right) = -2 \rightarrow \tan\left(\frac{1}{2}x\right) = -⅔$	**M1**	Attempt to obtain $\tan\left(\frac{1}{2}x\right) = k$ from $3\tan\left(\frac{1}{2}x\right) + 2 = 0$
	½x = −0.6 (– 0.588) → x = –1.2	**M1 A1**	$\tan^{-1} k$. Seeing ½x = – 33.69° or x= – 67.4° implies **M1M1**.
			Extra answers between -1.57 &1.57 lose the **A1**. Multiples of π are acceptable (eg – 0.374π)
	Total:	**3**	
10(ii)	$\frac{y+2}{3} = \tan\left(\frac{1}{2}x\right)$	**M1**	Attempt at isolating tan(½x)
	$\rightarrow f^{-1}(x) = 2\tan^{-1}\left(\frac{x+2}{3}\right)$	**M1 A1**	Inverse tan followed by × 2. Must be function of x for **A1**.
	−5,1	**B1 B1**	Values stated **B1** for -5, **B1** for 1.
	Total:	**5**	

Question	Answer	Marks	Guidance
10(iii)		**B1 B1 B1**	A tan graph through the first, third and fourth quadrants. (**B1**) An invtan graph through the first, second and third quadrants.(**B1**) Two curves clearly symmetrical about $y = x$ either by sight or by exact end points. Line not required. Approximately in correct domain and range. (Not intersecting.) (**B1**) Labels on axes not required.
	Total:	**3**	