

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3 (P3)

October/November 2017

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

- 1** Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

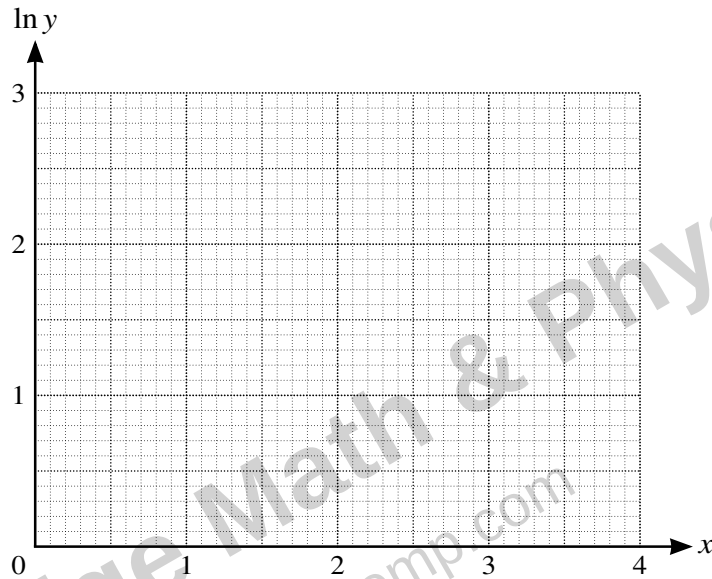
Cambridge Math & Physics

kambridgemp.com

- 2 Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y . The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a . Give your answers correct to 2 significant figures. [5]



.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 3** The equation $x^3 = 3x + 7$ has one real root, denoted by α .

(i) Show by calculation that α lies between 2 and 3.

[2]

Cambridge Math & Physics

kambridgemp.com

4 (i) Prove the identity $\tan(45^\circ + x) + \tan(45^\circ - x) \equiv 2 \sec 2x$. [4]

[4]

Cambridge Math & Physics

kanbridgemp.com

(ii) Hence sketch the graph of $y = \tan(45^\circ + x) + \tan(45^\circ - x)$ for $0^\circ \leq x \leq 90^\circ$.

[3]

Kambridge Math & Physics
kambridgemp.com

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

Cambridge Math & Physics
kamdridgemp.com

[illegible]

for $0 \leq x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

Cambridge Math & Physics
kcambridgemp.com

Kambridge Math & Physics
kambridgemp.com

- 7 (a) The complex number u is given by $u = 8 - 15i$. Showing all necessary working, find the two square roots of u . Give answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

Cambridge Math & Physics

kambridgemp.com

- (b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - i| \leq 2$ and $0 \leq \arg(z - i) \leq \frac{1}{4}\pi$. [4]

Kambridge Math & Physics
kambridgemp.com

8 Let $f(x) = \frac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}$.

(i) Express $f(x)$ in the form $A + \frac{B}{x+2} + \frac{C}{2x-1}$. [4]

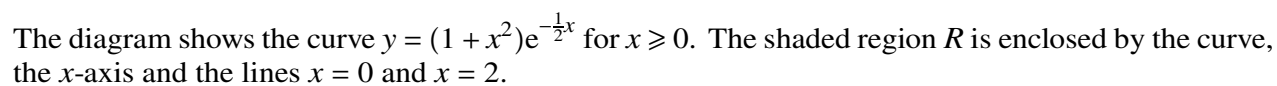
Cambridge Math & Physics

kambridgemp.com

(ii) Hence show that $\int_1^4 f(x) \, dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$. [5]

Kambridge Math & Physics

kambridgemp.com



- (i) Find the exact values of the x -coordinates of the stationary points of the curve.
- [4]
- Kambridge Math & Physics
kambridgemp.com

- (ii)** Show that the exact value of the area of R is $18 - \frac{42}{e}$.

[5]

Kambridge Math & Physics
kambridgemp.com

A blank sheet of white paper with ten horizontal dotted lines spaced evenly apart, intended for writing or drawing.

- 10 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

(i) Show that the lines do not intersect.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Calculate the acute angle between the directions of the lines.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Find the equation of the plane which passes through the point $(3, -2, -1)$ and which is parallel to both l and m . Give your answer in the form $ax + by + cz = d$. [5]

Cambridge Math & Physics

kambridgemp.com

Kambridge Math & Physics
kambridgemp.com

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.